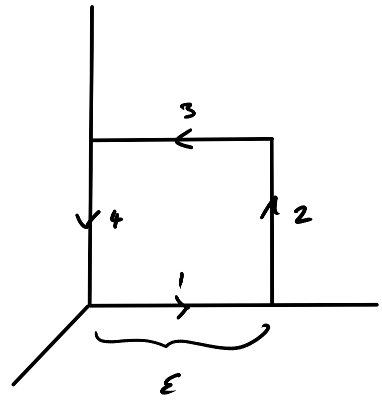
H 6.2



$$\vec{B}(0,y,z) = \vec{B}(0,0,0) + y \frac{\partial \vec{B}}{\partial y}\Big|_{\vec{r}=\vec{0}} + z \frac{\partial \vec{B}}{\partial y}\Big|_{\vec{r}=0}$$

$$\vec{F} = \oint dl\,(\vec{I} \times \vec{B}) \overset{zz}{=} (\vec{m} \times \vec{\nabla}) \times \vec{B}$$

R=9,5 0

1) $dl = dy \qquad \vec{I} = I\hat{e}_y$

$$\vec{B}(0,y,0) \approx \vec{B}(0,0,0) + y \frac{\partial B}{\partial y}\Big|_{\vec{r}=\vec{0}}$$

$$\vec{F}_1 = \int_0^{\varepsilon} dy\, I(\hat{e}_y \times \vec{B}) = I \int_0^{\varepsilon} dy \begin{pmatrix} B_z \\ 0 \\ -B_x \end{pmatrix}$$

$$= I \int_0^{\varepsilon} dy \begin{pmatrix} B_z(0) + y \frac{\partial B_z}{\partial y}(0) \\ 0 \\ -B_x(0) - y \frac{\partial B_x}{\partial y}(0) \end{pmatrix} = I\varepsilon \begin{pmatrix} B_z(0) + \frac{\varepsilon}{2} \frac{\partial B_z}{\partial y}(0) \\ 0 \\ B_x(0) - \frac{\varepsilon}{2} \frac{\partial B_x}{\partial y}(0) \end{pmatrix}$$

2) $dl = dz, \quad \vec{I} = I\hat{e}_z$

$$\vec{B}(0,\varepsilon,z) \approx B(0) + \varepsilon \frac{\partial B}{\partial y}(0) + z \frac{\partial B}{\partial z}(0)$$

$$F_2 = I \int_0^{\varepsilon} dy \begin{pmatrix} -B_y \\ B_x \\ 0 \end{pmatrix} \approx I\varepsilon \begin{pmatrix} -B_y(0) - \varepsilon \frac{\partial B_y}{\partial y}(0) - \frac{\varepsilon}{2} \frac{\partial B_y}{\partial z}(0) \\ B_x(0) + \varepsilon \frac{\partial B_x}{\partial y}(0) + \frac{\varepsilon}{2} \frac{\partial B_x}{\partial z}(0) \\ 0 \end{pmatrix}$$

3) und 4) so ähnlich.

$$\vec{F} = \sum_i F_i = I\varepsilon^2 \begin{pmatrix} -\frac{\partial B_z}{\partial z}(0) - \frac{\partial B_y}{} \\ \frac{\partial B_x}{\partial y}(0) \\ \frac{\partial B_x}{\partial z}(0) \end{pmatrix}$$

$$\vec{m} = |\vec{m}|\,\hat{e}_x$$

$$\left(\vec{m} \times \vec{\nabla}\right) \times \vec{B} = |\vec{m}| \begin{pmatrix} 0 \\ -\partial_z \\ \partial_y \end{pmatrix} \times \vec{B} = |\vec{m}| \begin{pmatrix} -\partial_z B_z - \partial_y B_y \\ \partial_y B_x \\ \partial_z B_x \end{pmatrix}$$

$$\approx m \begin{pmatrix} -\frac{\partial B_z}{\partial z}(0) - \frac{\partial B_y}{\partial y}(0) \\ \frac{\partial B_x}{\partial y}(0) \\ \frac{\partial B_x}{\partial z}(0) \end{pmatrix}$$

$$\vec{m} = \frac{1}{2} \int d^3r \left(\vec{r} \times \vec{j}(\vec{r})\right)$$

$$= \frac{1}{2} I \oint \vec{r} \times d\vec{r}$$

$$= \frac{1}{2} I \left[\int_0^{\varepsilon} \underbrace{\begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ dy \\ 0 \end{pmatrix}}_{0} + \int_0^{\varepsilon} \underbrace{\begin{pmatrix} 0 \\ \varepsilon \\ z \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ dz \end{pmatrix}}_{\begin{pmatrix} \varepsilon dz \\ 0 \\ 0 \end{pmatrix}} + \int_{\varepsilon}^{0} \underbrace{\begin{pmatrix} 0 \\ y \\ \varepsilon \end{pmatrix} \times \begin{pmatrix} 0 \\ dy \\ 0 \end{pmatrix}}_{\begin{pmatrix} -\varepsilon dy \\ 0 \\ 0 \end{pmatrix}} + \int_{\varepsilon}^{0} \underbrace{\begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ dz \end{pmatrix}}_{0} \right]$$

$$= I \varepsilon^2 \hat{e}_x$$