Analysis of π–π LQCD scattering data

Martin Ueding – mu@martin-ueding.de

2015-03-20

Contents of this presentation

Section 1

Data generation

Monte Carlo for quantum mechanics

Andreas Kell, Martin Efferz and Simon Blanke will introduce

- Feynman's path integral formalism
- Markov chains for weighted generation
- Metropolis algorithm
- Energy from correlation functions

Those ideas can be used for QCD

Lattice QCD

- Use action of QCD; parametrized by quark mass, . . .
- Generate configurations, weighted by $\exp(-S)$
- Examine observables on those configurations, meson operators in this case
- Correlation functions lead to masses

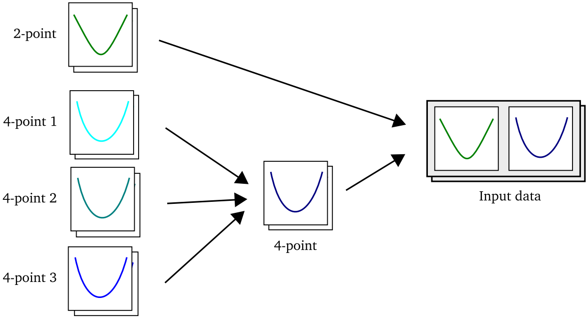
Section 2

Analysis methods

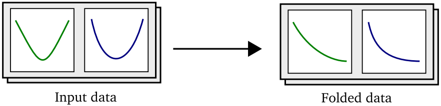
Subsection 1

Importing data

Input data



Folding



Subsection 2

Bootstrap

Alternatives?

Gaussian error propagation . . .

- is tedious
- assumes small errors
- does not scale

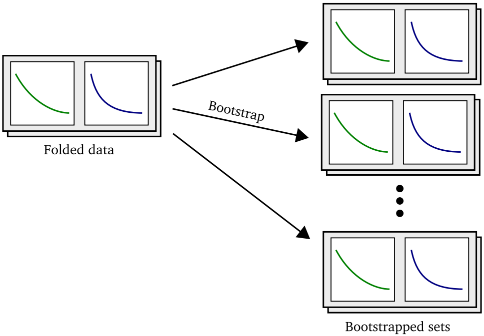
A quick review

The bootstrap method:

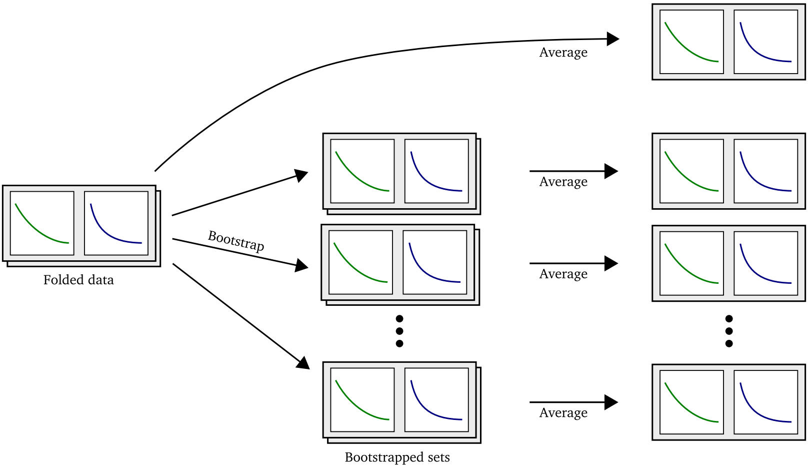
- Analysis function f: Transforms input data X into output data Y
  E.g. samples $X = \{x_i\}_i$ to median Y
- $f(X)$ is estimate for value
- Generate R samples from X: $\{\tilde{X}_i\}_i$
- Apply f to each sample: $\{f(\tilde{X}_i)\}_i$
- Error is standard deviation: $\Delta Y = \sigma\left(\{f(\tilde{X}_i)\}_i\right)$

Value and error without computing derivatives!

Generation of bootstrap samples



Averaging for further analysis



Subsection 3

Correlated fit

Correlation in data points

- Correlation functions are correlated in time
- Regular fit will give wrong χ^2 and $p \approx 1$
- Correlated fit is needed

New χ^2

New χ^2:

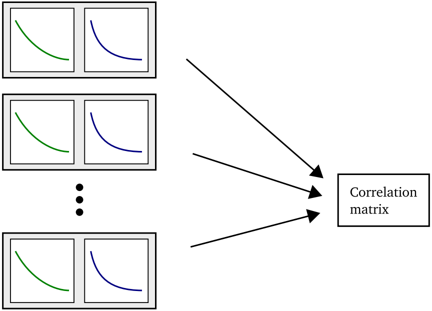
$$\chi^2 := \sum_{i,j}^{T} [\bar{x}_{iR} - f(t_i, \lambda)] \, C_{ij}^{-1} \left[\bar{x}_{jR} - f(t_j, \lambda)\right], \quad \bar{x}_{iR} := \frac{1}{R} \sum_{r=1}^{R} x_{ir}$$

Correlation matrix:

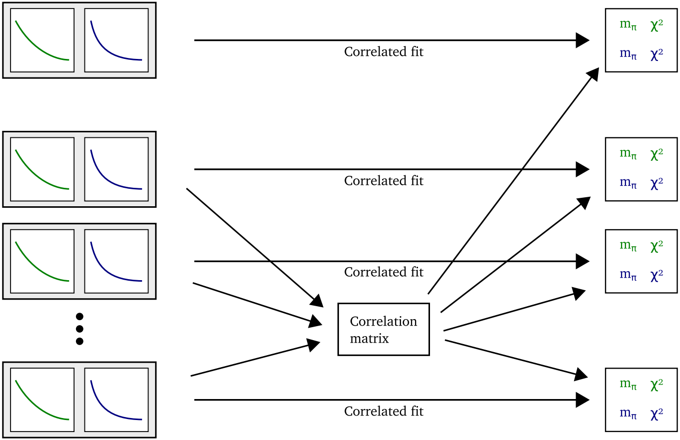
$$C_{ij} := \frac{1}{R[R-1]} \sum_{r=1}^{R} [x_{ir} - \bar{x}_{iR}][x_{jr} - \bar{x}_{jR}]$$

- λ Fit parameters
- i, j Time slice number
- R Number of bootstrap samples

Correlation matrix



Correlated fit



Subsection 4

Scattering length

From masses to scattering length

There is a relation between mass difference to scattering length (Lüscher 1986, (1.3)):

$$W = 2m - \frac{4\pi a_0}{mL^3}\left[1 + c_1 \frac{a_0}{L} + c_2 \frac{a_0^2}{L^2}\right]$$

- W Mass of π-π-system
- m Mass of single π
- a_0 s-wave scattering length
- L Number of spatial lattice sites
- c_1 $-2{,}837\,297$
- c_2 $6{,}375\,183$

Motivation of Lüscher's formula

- Two identical particles in L^3 box
- $H = H_0 + V$, V short ranged
- Probability to be in interaction range $\propto L^{-3}$
- First order of V: Energy shift $\propto L^{-3}$
- Born series: $V(\mathbf{0}, \mathbf{0}) \propto$ scattering length
- Higher order in V gives L^{-4} and higher terms

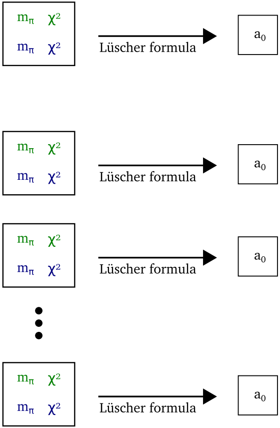
Application of Lüscher's formula

Solve for a_0

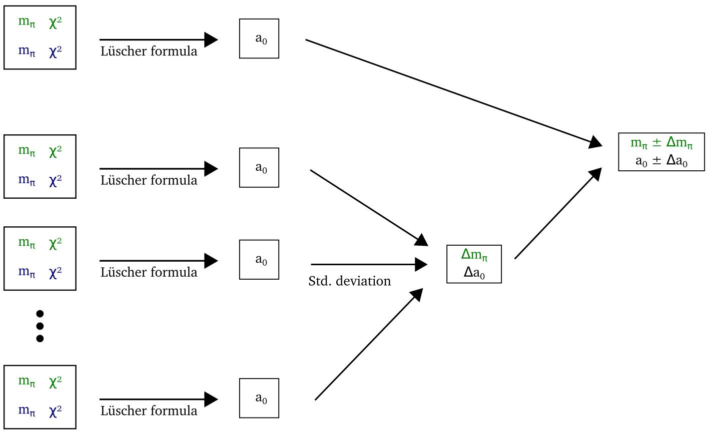
$$2m - W - \frac{4\pi a_0}{mL^3}\left[1 + c_1\frac{a_0}{L} + c_2\frac{a_0^2}{L^2}\right] = 0$$

using root finding like

- Newton from $a_0 = 0$
- Brent (1973)



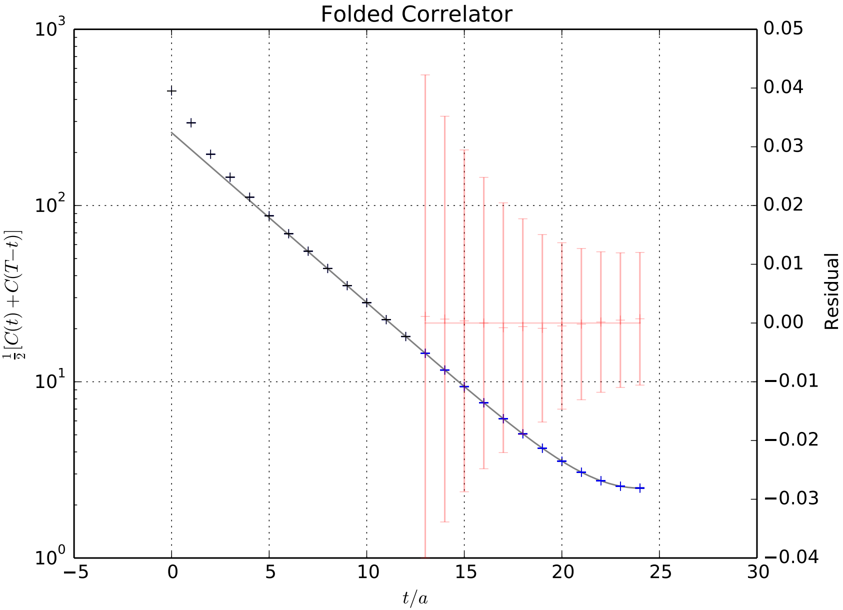
End results



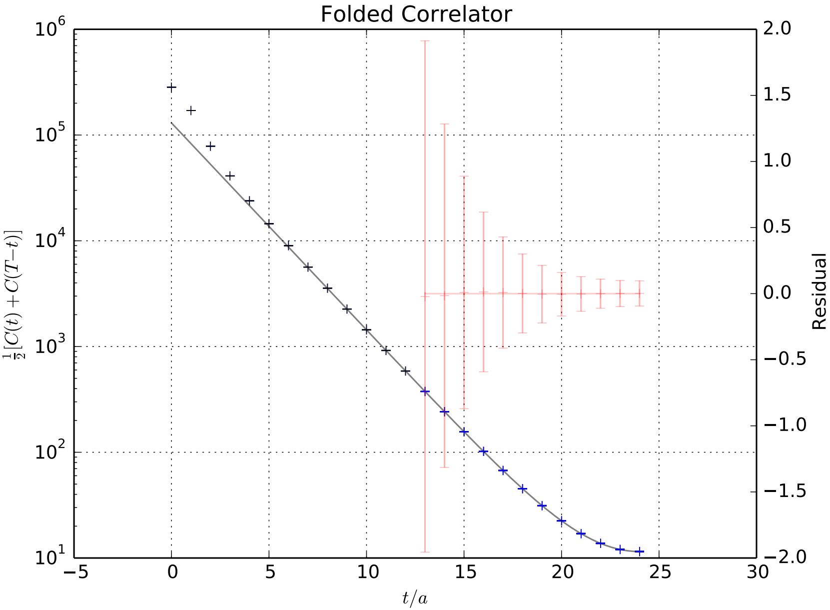
mπ χ²
mπ χ²
Lüscher formula
a0
Std. deviation
Δmπ
Δa0
mπ ± Δmπ
a0 ± Δa0

Section 3

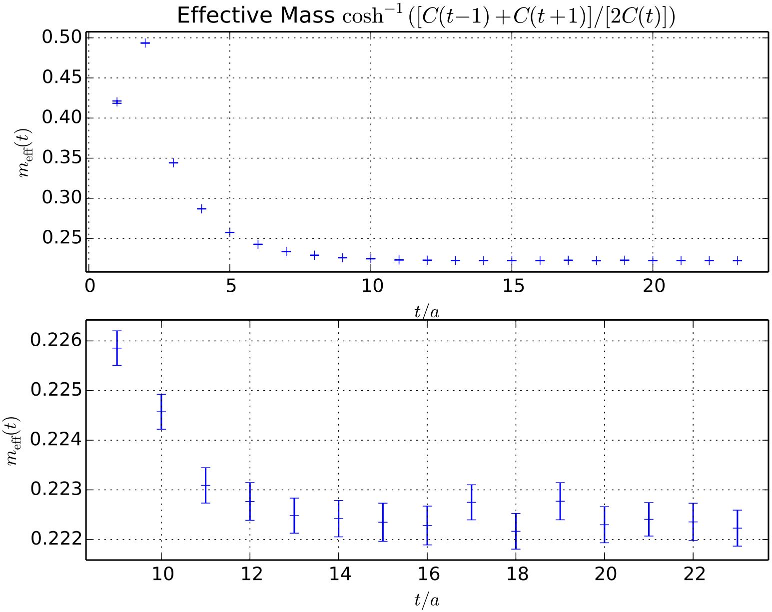
Results

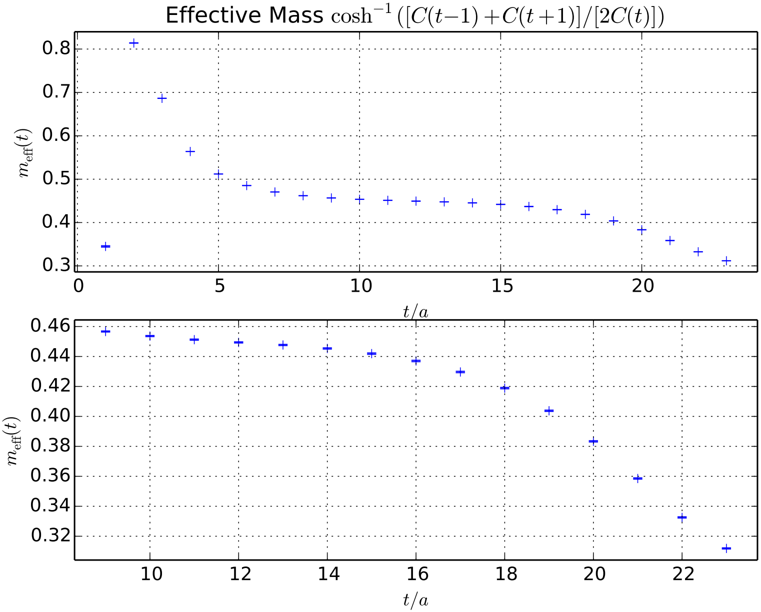


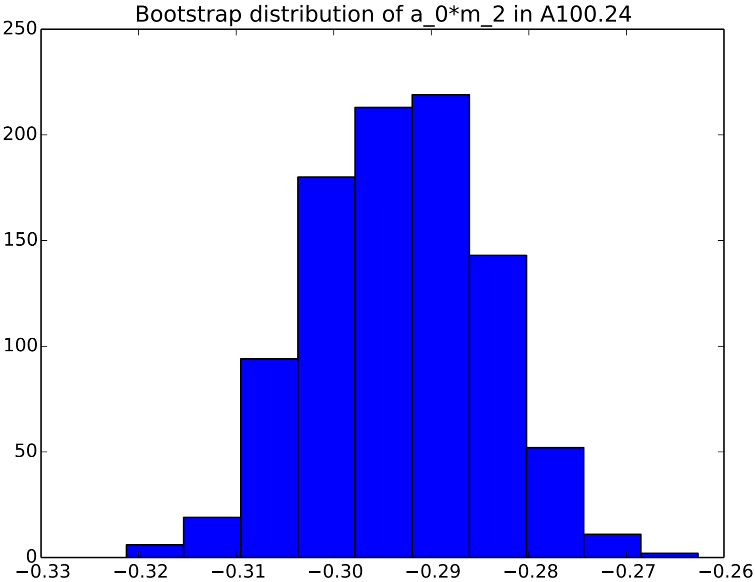
Folded Correlator
10^3
10^2
10^1
10^0
$\frac{1}{2}[C(t)+C(T-t)]$
−5
0
5
10
15
20
25
30
t/a
0.05
0.04
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
−0.04
Residual

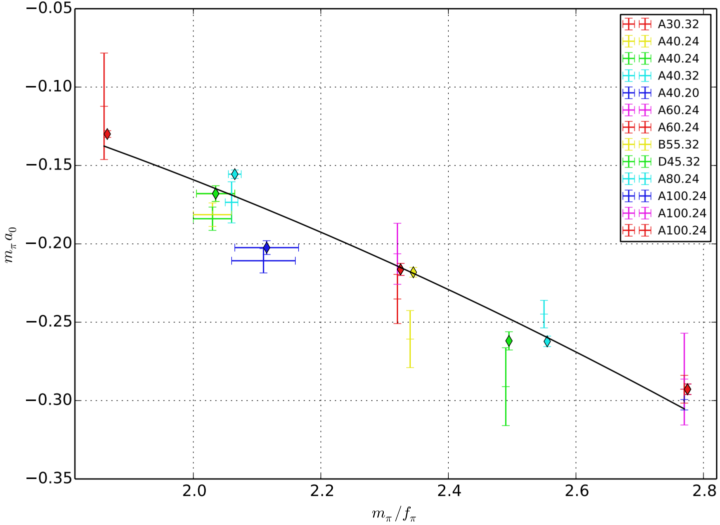


Folded Correlator
$\frac{1}{2}[C(t)+C(T-t)]$
10^6
10^5
10^4
10^3
10^2
10^1
−5
0
5
10
15
20
25
30
t/a
Residual
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0


Effective Mass $\cosh^{-1}([C(t-1)+C(t+1)]/[2C(t)])$
0.50
0.45
0.40
0.35
0.30
0.25
$m_{\text{eff}}(t)$
0
5
10
15
20
t/a
0.226
0.225
0.224
0.223
0.222
$m_{\text{eff}}(t)$
10
12
14
16
18
20
22
t/a


Effective Mass $\cosh^{-1}([C(t-1)+C(t+1)]/[2C(t)])$
$m_{\text{eff}}(t)$
0.8
0.7
0.6
0.5
0.4
0.3
0
5
10
15
20
t/a
$m_{\text{eff}}(t)$
0.46
0.44
0.42
0.40
0.38
0.36
0.34
0.32
10
12
14
16
18
20
22
t/a


Bootstrap distribution of a_0*m_2 in A100.24
250
200
150
100
50
0
−0.33
−0.32
−0.31
−0.30
−0.29
−0.28
−0.27
−0.26



Diamond data points from draft paper, slightly shifted.

Pion decay constants from (Helmes et al. 2014, table 1).

References

π–π Scattering with $N_f = 2 + 1 + 1$ Twisted Mass Fermions (2014). Bonn, Germany. arXiv: 1412.0408 [hep-lat].

Lüscher, M. (1986). “Volume Dependence of the Energy Spectum in Massive Quantum Field Theories”. In: *Commun. Math. Phys.* 105, pp. 153–188.

Get the paper

To get the paper and slides, go to:

- martin-ueding.de
- University
- Master of Science in Physics
- physics760
  Computational Physics

Or scan the code:

Made with LaTeX Beamer, SciPy, matplotlib and Inkscape.