Disclaimer

This is a problem set (as turned in) for the module physics751.

This problem set is not reviewed by a tutor. This is just what I have turned in.

All problem sets for this module can be found at
http://martin-ueding.de/de/university/msc_physics/physics751/.

[disclaimer]

physics751 – Group Theory

Problem Set 13

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2015-02-02

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1 Tensor products of Young tableaux

1.1

Problem statement

Calculate $\bar{3} \otimes \bar{3}$ and $\bar{3} \otimes \bar{3} \otimes \bar{3}$.

To solve arbitrary tensor products I distilled the algorithm given on the problem set into an interactive website which can be found at:

`http://app.martin-ueding.de/young-frame-product/`

Or just scan the following QR code:

1.1.1 Calculation of $\bar{3} \otimes \bar{3}$

Using my program, I get the following:

$$\ydiagram{1,1} \otimes \ydiagram{1,1} = \ydiagram{2,2} \oplus \ydiagram{2,1,1} = \ydiagram{2,2} \oplus \ydiagram{1}.$$

In the last step, it is used that $N = 3$ and the first column can therefore be omitted. This can also be rewritten in terms of the dimensions:

$$\bar{3} \otimes \bar{3} = 6 \oplus 3.$$

1.1.2 Calculation of $\bar{3} \otimes \bar{3} \otimes \bar{3}$

Now this has to be multiplied with $\bar{3}$ again. First, I expand this:

$$\left[\ydiagram{2,2} \oplus \ydiagram{1}\right] \otimes \ydiagram{1,1} = \ydiagram{2,2} \otimes \ydiagram{1,1} \oplus \ydiagram{1} \otimes \ydiagram{1,1}$$

Now I use my program to compute the products of this. I will put brackets around the summands of the above equation make it clear which belongs to which.

$$= \left[\ydiagram{3,3} \oplus \ydiagram{3,2,1}\right] \oplus \left[\ydiagram{2,1} \oplus \ydiagram{1,1,1}\right]$$

The second summand can be simplified and I am left with:

$$= \ydiagram{3,3} \oplus 2\,\ydiagram{2,1} \oplus \ydiagram{1,1,1}$$

The program also gives the dimensions, so I can write this as:

$$= 10 \oplus 8^2 \oplus 1$$

The total dimension is 27, so this checks out.

1.2

Problem statement

Calculate $8 \otimes 8$.

The expected dimension would be 64. This is the output of the program:

$$\ydiagram{2,1} \otimes \ydiagram{2,1} = \ydiagram{4,2} \oplus \ydiagram{4,1,1} \oplus \ydiagram{3,3} \oplus 2\,\ydiagram{3,2,1} \oplus \ydiagram{2,2,2}$$

The program then computed the dimensions and gets:

$$27 \oplus 10 \oplus 10 \oplus 8^2 \oplus 1$$

This sum is 64, so it matches the dimensions.

1.3 Exact SU(3) symmetry

In the presence problem A 13.3 we looked at the scattering of two open charm mesons with each other. Because we assumed the exact SU(3) symmetry and there are three of those D quarks, we need a three dimensional representation. The one that applied here is the Γ_3 one. Then we took $3 \otimes 3 = 6 \oplus \bar{3}$ and said that there are two irreducible representations, therefore there are two independent scattering amplitudes.

Side question

Did I say this correctly?

Now we scatter one of those D quarks with anti D quarks. There is another three dimensional representation, the $\Gamma_{\bar{3}}$. With this, I can do the same calculation:

$$3 \otimes \bar{3} = 8 \oplus 1, \qquad \ydiagram{1} \otimes \ydiagram{1,1} = \ydiagram{2,1} \oplus \ydiagram{1,1,1}$$

So there are two irreducible representations here as well. Therefore, there are two scattering amplitudes as well.