Disclaimer

This is a reviewed problem set for the module physics751.

This problem set was reviewed by a tutor. *This does not mean that it is a perfect solution. Neither I or the tutor imply that there are no further mistakes in this document.*

All problem sets for this module can be found at
http://martin-ueding.de/de/university/msc_physics/physics751/.

[disclaimer]

Group Theory 10 — Martin Ueding — 2014-12-17

Part (a)

From the problem set:

> A and B letters are designated for 1-dimensional IRs. A is used if rotations about the principal axis is symmetric, and B if rotations about the principal axis is anti-symmetric.

v_1 and v_2 are symmetric in C_2. v_3 is antisymmetric in C_2. Seems to fit.

Part (b)

Construct the matrices in that basis.

$$D(E) = 1\!\!1_6$$

$$D(C_2) = \begin{pmatrix} & 1 & & & & \\ 1 & & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & 1 \end{pmatrix} \qquad D(\sigma_v) = \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -1 & \\ & & & & & 1 \end{pmatrix}$$

$$D(\sigma_{v'}) = \begin{pmatrix} & 1 & & & & \\ 1 & & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix}$$

Now the characters:

$$\chi(E) = 6 \qquad \chi(C_2) = 0 \qquad \chi(\sigma_v) = 4 \qquad \chi(\sigma_{v'}) = 2$$

Part (c)

$$g_{A_1} = \frac{1}{4}\left[1 \cdot 6 + 1 \cdot 0 + 1 \cdot 4 + 1 \cdot 2\right] = \frac{12}{4} = 3$$

$$g_{A_2} = \frac{1}{4}\left[1\cdot 6 + 1\cdot 0 - 1\cdot 4 - 1\cdot 2\right] = 0 \quad ✓$$

$$g_{B_1} = \frac{1}{4}\left[1\cdot 6 + 1\cdot 4 - 1\cdot 2\right] = \frac{8}{4} = 2 \quad ✓$$

$$g_{B_2} = \frac{1}{4}\left[1\cdot 6 - 1\cdot 4 + 1\cdot 2\right] = \frac{4}{4} = 1 \quad ✓$$

$3A_1 \oplus 2B_1 \oplus B_2$ ✓ checks out.

Part (d)

$$P_{A_1} = \frac{1}{4}\left[D(E) + D(C_2) + D(C_v) + D(C_{v'})\right]$$

$$P_{B_1} = \frac{1}{4}\left[D(E) - D(C_2) + D(C_v) - D(C_{v'})\right]$$

$$P_{B_2} = \frac{1}{4}\left[D(E) - D(C_2) - D(C_v) + D(C_{v'})\right]$$

Now plug the matrices in.

$$P_{A_1} = \frac{1}{4}\left[\begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix} + \begin{pmatrix} & 1 & & & & \\ 1 & & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & 1 \end{pmatrix} + \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -1 & \\ & & & & & 1 \end{pmatrix} + \begin{pmatrix} & 1 & & & & \\ 1 & & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix}\right]$$

$$= \frac{1}{4}\begin{pmatrix} 2 & 2 & & & & \\ 2 & 2 & & & & \\ & & 4 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 4 \end{pmatrix}$$

$$P_{B_1} = \frac{1}{4}\left[\begin{pmatrix} 1 &&&&& \\ & 1 &&&& \\ && 1 &&& \\ &&& 1 && \\ &&&& 1 & \\ &&&&& 1 \end{pmatrix} - \begin{pmatrix} & 1 &&&& \\ 1 &&&&& \\ && 1 &&& \\ &&& -1 && \\ &&&& -1 & \\ &&&&& 1 \end{pmatrix} + \begin{pmatrix} 1 &&&&& \\ & 1 &&&& \\ && 1 &&& \\ &&& 1 && \\ &&&& -1 & \\ &&&&& 1 \end{pmatrix} - \begin{pmatrix} & 1 &&&& \\ 1 &&&&& \\ && 1 &&& \\ &&& -1 && \\ &&&& 1 & \\ &&&&& 1 \end{pmatrix}\right]$$

$$= \frac{1}{4}\begin{pmatrix} 2 & -2 &&&& \\ -2 & 2 &&&& \\ && 0 &&& \\ &&& 4 && \\ &&&& 0 & \\ &&&&& 0 \end{pmatrix}$$

$$P_{B_2} = \frac{1}{4}\left[\begin{pmatrix} 1 &&&&& \\ & 1 &&&& \\ && 1 &&& \\ &&& 1 && \\ &&&& 1 & \\ &&&&& 1 \end{pmatrix} - \begin{pmatrix} & 1 &&&& \\ 1 &&&&& \\ && 1 &&& \\ &&& -1 && \\ &&&& -1 & \\ &&&&& 1 \end{pmatrix} - \begin{pmatrix} 1 &&&&& \\ & 1 &&&& \\ && 1 &&& \\ &&& 1 && \\ &&&& -1 & \\ &&&&& 1 \end{pmatrix} + \begin{pmatrix} & 1 &&&& \\ 1 &&&&& \\ && 1 &&& \\ &&& -1 && \\ &&&& 1 & \\ &&&&& 1 \end{pmatrix}\right]$$

$$= \frac{1}{4}\begin{pmatrix} 0 & 0 &&&& \\ 0 & 0 &&&& \\ && 0 &&& \\ &&& 0 && \\ &&&& 4 & \\ &&&&& 0 \end{pmatrix}$$

Then apply those to the state

$$F = \begin{pmatrix} SH1 \\ SH2 \\ SO \\ Px \\ Py \\ Pz \end{pmatrix}.$$

$$P_{A_1}F = \frac{1}{4}\begin{pmatrix} 2 & 2 &&&& \\ 2 & 2 &&&& \\ && 4 &&& \\ &&& 0 && \\ &&&& 0 & \\ &&&&& 4 \end{pmatrix}\begin{pmatrix} SH1 \\ SH2 \\ SO \\ Px \\ Py \\ Pz \end{pmatrix} = \begin{pmatrix} [SH_1 + SH_2]/2 \\ [SH_1 + SH_2]/2 \\ SO \\ 0 \\ 0 \\ Pz \end{pmatrix}$$

$$P_{B_1}F = \frac{1}{4}\begin{pmatrix} 2 & -2 & & & & \\ -2 & 2 & & & & \\ & & 0 & & & \\ & & & 4 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}\begin{pmatrix} SH1 \\ SH2 \\ SO \\ Px \\ Py \\ Pz \end{pmatrix} = \begin{pmatrix} [SH_1 - SH_2]/2 \\ [SH_2 - SH_1]/2 \\ 0 \\ Px \\ 0 \\ 0 \end{pmatrix}$$

$$B_2 F = \frac{1}{4}\begin{pmatrix} 0 & 0 & & & & \\ 0 & 0 & & & & \\ & & 0 & & & \\ & & & 0 & & \\ & & & & 4 & \\ & & & & & 0 \end{pmatrix}\begin{pmatrix} SH1 \\ SH2 \\ SO \\ Px \\ Py \\ Pz \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ Py \\ 0 \end{pmatrix}$$