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This is a reviewed problem set for the module physics751.

This problem set was reviewed by a tutor. *This does not mean that it is a perfect solution. Neither I or the tutor imply that there are no further mistakes in this document.*

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[disclaimer]

physics751 – Group Theory

# Problem Set 9


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## 1 Charge operator

**Problem statement**

Determine the expectation values of $\hat{q}(3)$.

The correct normalization that should have been calculated in the presence problems is given by a factor $1/\sqrt{2}$.

$$\left\langle \Psi^{[M,\overset{A}{S}]}_{\text{proton}} \middle| \hat{q}(3) \middle| \Psi^{[M,\overset{A}{S}]}_{\text{proton}} \right\rangle = \frac{1}{2} \langle \text{udu} - \text{duu} \,|\, \hat{q}(3) \,|\, \text{udu} - \text{duu} \rangle$$

I use the linearity of the scalar product. I further assume that any flavor states that are not exactly the same are orthogonal. Therefore, only those elements remain:

$$= \frac{1}{2} \langle \text{udu} \,|\, \hat{q}(3) \,|\, \text{udu} \rangle + \frac{1}{2} \langle \text{duu} \,|\, \hat{q}(3) \,|\, \text{duu} \rangle .$$

Now I apply the charge operator and obtain

$$= \frac{1}{2} \left\langle \text{udu} \middle| \frac{2}{3} \middle| \text{udu} \right\rangle + \frac{1}{2} \left\langle \text{duu} \middle| \frac{2}{3} \middle| \text{duu} \right\rangle .$$

I pull the fraction out front.

$$= \frac{1}{3} \langle \text{udu} \,|\, \text{udu} \rangle + \frac{1}{3} \langle \text{duu} \,|\, \text{duu} \rangle$$

Those scalar products are just unity, so only

$$= \frac{2}{3}$$

remains.

For the second state, the normalization is $1/\sqrt{6}$. The matrix element is:

$$\left\langle \Psi^{[M,\overset{S}{\cancel{A}}]}_{\text{proton}} \middle| \hat{q}(3) \middle| \Psi^{[M,\overset{S}{\cancel{A}}]}_{\text{proton}} \right\rangle = \frac{1}{6} \langle 2\text{uud} - \text{udu} - \text{duu} | \hat{q}(3) | 2\text{uud} - \text{udu} - \text{duu} \rangle$$

I use the same orthogonality and linearity here as well.

$$= \frac{1}{6} \langle 2\text{uud} | \hat{q}(3) | 2\text{uud} \rangle + \frac{1}{6} \langle \text{udu} | \hat{q}(3) | \text{udu} \rangle + \frac{1}{6} \langle \text{duu} | \hat{q}(3) | \text{duu} \rangle$$

I now apply the charge operator to the last quark.

$$= \frac{1}{6} \left\langle 2\text{uud} \middle| -\frac{1}{3} \middle| 2\text{uud} \right\rangle + \frac{1}{6} \left\langle \text{udu} \middle| \frac{2}{3} \middle| \text{udu} \right\rangle + \frac{1}{6} \left\langle \text{duu} \middle| \frac{2}{3} \middle| \text{duu} \right\rangle$$

The first scalar product is 4, the last two are 1. This leaves

$$\begin{aligned} &= \frac{1}{6} \left[ -\frac{4}{3} + \frac{2}{3} + \frac{2}{3} \right] \\ &= 0. \end{aligned}$$

## 2 Spin operator

**Problem statement**

Repeat the previous calculations with the spin operator $\hat{\sigma}_3$ with eigenvalues $\pm 1$.

$$\begin{aligned} \left\langle \Psi^{[M,\overset{A}{\cancel{S}}]}_{\text{proton}} \middle| \hat{q}(3) \middle| \Psi^{[M,\overset{A}{\cancel{S}}]}_{\text{proton}} \right\rangle &= \frac{1}{2} \langle \text{udu} - \text{duu} \,|\, \hat{\sigma}_3 \,|\, \text{udu} - \text{duu} \rangle \\ &= \frac{1}{2} \langle \text{udu} \,|\, \hat{\sigma}_3 \,|\, \text{udu} \rangle + \frac{1}{2} \langle \text{duu} \,|\, \hat{\sigma}_3 \,|\, \text{duu} \rangle \\ &= \frac{1}{2} \langle \text{udu} \,|\, 1 \,|\, \text{udu} \rangle + \frac{1}{2} \langle \text{duu} \,|\, 1 \,|\, \text{duu} \rangle \\ &= 1 \end{aligned}$$

please write the spin wave function like $\frac{1}{\sqrt{2}} |\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\rangle$ to not confuse it with the flavor wave function

$$
\begin{aligned}
\left\langle \Psi_{\text{proton}}^{[M,\overset{S}{\cancel{A}}]} \middle| \hat{q}(3) \middle| \Psi_{\text{proton}}^{[M,\overset{S}{\cancel{A}}]} \right\rangle &= \frac{1}{6} \langle 2\text{uud} - \text{udu} - \text{duu} | \hat{\sigma}_3 | 2\text{uud} - \text{udu} - \text{duu} \rangle \\
&= \frac{1}{6} \langle 2\text{uud} | \hat{\sigma}_3 | 2\text{uud} \rangle + \frac{1}{6} \langle \text{udu} | \hat{\sigma}_3 | \text{udu} \rangle + \frac{1}{6} \langle \text{duu} | \hat{\sigma}_3 | \text{duu} \rangle \\
&= \frac{1}{6} \langle 2\text{uud} \,|\, -1 \,|\, 2\text{uud} \rangle + \frac{1}{6} \langle \text{udu} \,|\, 1 \,|\, \text{udu} \rangle + \frac{1}{6} \langle \text{duu} \,|\, 1 \,|\, \text{duu} \rangle \\
&= \frac{1}{6} [-4 + 1 + 1] \\
&= -\frac{1}{3}
\end{aligned}
$$