A 9.1

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Part 1

Confirm

$$Y^{[2,1]}_{121,121} = e + (13) - (12) - (123).$$

$[2,1] \triangleq \begin{array}{|c|c|}\hline & \\ \hline \\ \cline{1-1}\end{array}$. Then fill in 121 and get $\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array}$.

Now $Y^{[2,1]}_{121} = P^{[2,1]}_{121} \; Q^{[2,1]}_{121}$

Compute horizontal subgroups for $P^{\cdots}_{\cdots}$:

$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array} \rightsquigarrow \begin{array}{c}\{e, (13)\} \\ \{e\}\end{array} = e + (13)$$

Now vertical subgroups for $Q^{\cdots}_{\cdots}$:

$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array} \rightsquigarrow \{e, (12)\} \text{ and } \{e\}. = [e - (12)]e = e - (12)$$

Now form the product.

$$\begin{aligned}[e + (13)][e - (12)] &= e - (12) + (13) - (13)(12) \\ &= e - (12) + (13) - (123)\end{aligned}$$

Okay, this checks out. I will skip the next part because I expect this to work similarly. Next is the transfer permutation.

$$\sigma_{211,121}\, Y^{[2,1]}_{121,121}\, \sigma_{121,211} = (23)\left[e + (13) - (12) - (123)\right](23)^{-1}$$
$$= \left[(23) + (123) - (132) - (13)\right](23)$$
$$= e + (12) - (13) - (132) = Y^{[2,1]}_{211}$$

This works.

Part 2

It looks like a straight application of cycle multiplication. I will skip this, I would rather be interested in the meaning of those terms.