A 8.1

Part 1

$$g \sim h \iff \exists i \in G: \quad i g i^{-1} = h$$

$$ig = hi$$

$$\{ i g i^{-1} j h j^{-1}, g \in k_a, h \in k_b, i, j \in G \} = \{ j h j^{-1} i g i^{-1}, \dots \}$$

Solution:

$$Cl_i \, Cl_j = \{ g_a^i \, g_b^j \mid g_a^i \in Cl_i, \, g^j \in Cl_j \}$$

$$= \{ g_b^j \underbrace{g_b^{j^{-1}} g_a^i g_b^j}_{\in Cl_i} \mid g_a^i \in Cl_i, \, g^j \in Cl_j \}$$

$$= \{ g_b^j \, g_a^i \mid g_a^i \in Cl_i, \, g^j \in Cl_j \}$$

$$= Cl_j \, Cl_i$$

Part 2

$c_1 = \{e\}$, $c_2 = \{c, c^3\}$, $c_3 = \{c^2\}$, $c_4 = \{b, bc^2\}$,
$c_5 = \{bc, bc^3\}$

$$c_1 c_2 = \{c, c^3\} = c_2$$

$$c_2 c_2 = \{c^2, c^4, c^6\} = \{e, c^2\} = c_1 \cup c_2$$

$$c_2 c_3 = \{c^3, c^5\} = \{c^1, c^3\} = c_2$$

$$C_4 C_2 = \{ bc,\ bc^3,\ bc^3,\ bc^5 \} = C_5$$

$$C_5 C_2 = \{ bc^2,\ bc^4,\ bc^4,\ bc^6 \} = \{ b,\ bc^2 \} = C_4$$

$$C_3 C_3 = C_1$$

$$C_4 C_3 = \{ bc^2,\ bc^4 \} = C_4$$

$$C_5 C_3 = C_5$$

$$C_4 C_4 = \{ b^2,\ bc^2bc^2,\ bbc^2 \} = \{ e,\ c^2,\ bbc^3c^3c^2 \}$$

$$= \{ e,\ c^2,\ e \} = e_1 \cup C_3$$

$$C_4 C_5 = \{ b^2c,\ bbc^3,\ bc^2bc,\ bc^2bc^3 \}$$

$$= \{ c,\ c^3,\ bbc^3c^3c,\ bbc^3c^3c^3 \}$$

$$= \{ c,\ c^3,\ c^9,\ c^7 \} = \{ c,\ c^3,\ c^3 \} = C_2$$

$$C_5 C_5 = \{ bcbc,\ bcbc^3,\ bc^3bc,\ bc^3bc^3 \}$$

$$= \{ bbc^3c,\ bbc^6,\ bbc^3c^3c^3c,\ bbc^3c^3c^3c^3 \}$$

$$= \{ e,\ c^2,\ c^2,\ e \} = \underline{2}\, C_1 + \underline{2}\, C_3$$

Multiples have to be taken into account, like was done in the last example only. They are not really written as unions, but as a sort of +.