A5.1

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Part 1

$$D^G(g_1)\, D^G(g_2) \overset{\text{def}}{=} D^{G/N}(g_1 N)\, D^{G/N}(g_2 N)$$

$$= D^{G/N}(g_1 N g_2 N) = D^{G/N}(g_1 g_2 N)$$

(coset multiplication)

$$= D^G(g_1 g_2) \quad (\text{def})$$

The kernel is N, so it is not faithful.

Part 2

(a)

$$g_1 g_2 \hat{=} D(g_1 g_2)^\dagger$$

$$= [D(g_1)\, D(g_2)]^\dagger$$

$$= D(g_2)^\dagger D(g_1)^\dagger$$

$$\hat{=} g_2 g_1$$

$\Rightarrow$ (a) is not a representation

(b)

$$g_1 g_2 \hat{=} D(g_2^{-1} g_1^{-1})^\dagger$$

$$= D(g_1^{-1})^\dagger D(g_2^{-1})^\dagger$$

$$\hat{=} g_1 g_2$$

$\Rightarrow$ (b) is a representation.

(c)

$$g_1 g_2 \hat{=} \det(D(g_1 g_2))$$

$$= \det(D(g_1)\, D(g_2))$$

$$= \det(D(g_1))\, \det(D(g_2))$$

$$\hat{=} g_1 g_2$$

$\Rightarrow$ (c) is a representation, although with scalar.

(d)

$$g_1 g_2 \hat{=} \mathrm{Tr}(D(g_1 g_2))$$

$$= \mathrm{Tr}(D(g_1)\, D(g_2))$$

Is that a sum or so? Can this be simplified?

Apparently not $\Rightarrow$ not a representation.