

Part 1

$$C_2 = \{e, c\} \quad c^2 = e$$

$$T = \begin{pmatrix} e & c \\ c & e \end{pmatrix}$$

Part 2

$$C_3 = \{e, c, c^2\} \quad c^3 = e$$

$$T = \begin{pmatrix} e & c & c^2 \\ c & c^2 & e \\ c^2 & e & c \end{pmatrix}$$

Part 3

$$D_2 = \{e, c, b, bc\} \quad c^2 = b^2 = (bc)^2 = e$$

$$T = \begin{pmatrix} ee & ce & be & bce \\ ec & cc & bc & bcc \\ eb & cb & bb & bcb \\ ebc & cbc & bbc & bcbc \end{pmatrix}^T$$

$$\begin{aligned} cb &= bc \\ c^2b &= cbc \\ b &= cbc \\ b^2 &= bcbc \quad \checkmark \end{aligned}$$

$$= \begin{pmatrix} e & c & b & bc \\ c & e & bc & b \\ b & bc & e & c \\ bc & b & c & e \end{pmatrix}^T$$

Part 4

$$D_3 = \{e, c, c^2, b, bc, bc^2\} \quad c^3 = b^2 = (bc)^2 = e$$

$$T = \begin{pmatrix} ee & ec & ec^2 & eb & ebc & ebc^2 \\ ce & cc & cc^2 & cb & cbc & cbc^2 \\ c^2e & c^2c & c^2c^2 & c^2b & c^2bc & c^2bc^2 \\ be & bc & bc^2 & bb & bbc & bbc^2 \\ bce & bcc & bcc^2 & bcb & bc bc & bc bc^2 \\ bc^2e & bc^2c & bc^2c^2 & bc^2b & bc^2bc & bc^2bc^2 \end{pmatrix}$$

$$= \begin{pmatrix} e & c & c^2 & b & bc & bc^2 \\ c & c^2 & e & \underline{cb} & \underline{cbc} & \underline{cbc^2} \\ c^2 & e & c & c^2b & c^2bc & c^2bc^2 \\ b & bc & bc^2 & e & c & c^2 \\ bc & bc^2 & b & bcb & e & bc bc^2 \\ bc^2 & b & bc & bc^2b & bc^2bc & bc^2bc^2 \end{pmatrix}$$

/// $cbc = b^{-1}bc bc = b^{-1} = b$

$bcb = bbc^2 = c^2$

/// $cbc^2 = cbc c = bc$

$bccb = bc bc^2 = c$

/// $cb = b^{-1}bc bc c^{-1} = b^{-1}c^{-1} = bc^2$

$ccb = c bc^2 = bc^2 c^2 = bc$

$$= \begin{pmatrix} e & c & c^2 & b & bc & bc^2 \\ c & c^2 & e & bc^2 & b & bc \\ c^2 & e & c & bc & bc^2 & b \\ b & bc & bc^2 & e & c & c^2 \\ bc & bc^2 & b & c^2 & e & c \\ bc^2 & b & bc & c & c^2 & e \end{pmatrix}$$