

Part 1

We have $eg = g$.

$$\underbrace{g^{-1}eg}_{\text{must be } g^{-1}} = e$$

must be g^{-1} to work out.
 $\Rightarrow g^{-1}e = g^{-1}$.

Part 2

Let a be another inverse

$$a = g^{-1}ga = g^{-1} \Rightarrow \text{unique.}$$

But what about right-left?

$$g^{-1}g = e$$

Let $a = g^{-1}$ for right side.

$$ga = e$$

$$g^{-1}ga = g^{-1}e$$

$$a = g^{-1}$$