Disclaimer

This is a problem set (as turned in) for the module physics7501.

This problem set is not reviewed by a tutor. This is just what I have turned in.

All problem sets for this module can be found at
http://martin-ueding.de/de/university/msc_physics/physics7501/.

[disclaimer]

physics7501 – Advanced Quantum Field Theory

Problem Set 5

Martin Ueding

mu@martin-ueding.de

2015-11-24

Tutor: Thorsten Schimannek

Problem	Achieved points	Possible points
Yukawa corrections		20
Total		20

This document consists of 3 pages.

I am not sure whether I got all the renormalization straight so far. When we first started with Feynman diagrams it seemed that there was no need for those field strength renormalization factors Z at all. To sort those things out I would like to write down what I think I have understood such that you can point to where my misunderstanding starts.

The S-matrix translates states from the past infinity to the future infinity. We use that for scattering as we assume that the time of the scattering is limited and the particles did not interact with each other before the time of the scattering. The assumption of non-interaction between the distinct particles seems reasonable as they are far away from each other. The propagation from the past infinity to the scattering was somewhat excluded as we only looked into amputated diagrams. The amputated diagrams could be computed with the Feynman rules and the various tricks that we have learned during the course so far.

I assume that taking amputated diagrams means to approximate the time evolution from past infinity to the scattering with the free theory. If that is indeed the case, we only made an approximation there such that the S-matrix would be easier to compute. The disconnected diagrams (the vacuum bubbles) factored out of the correlation functions and therefore do not contribute at all. I took the latter statement as an exact statement (the proof looked exact). Perhaps I have misunderstood the amputated diagrams as being an analogous thing and thought that this was not an approximation.

From the charge and mass renormalization I have the impression that the amputation is an approximation that we would like to get rid of in Chapter 7 of *the book*. The propagation from past infinity (and to future infinity) is also perturbed by the various interactions like the electron self-energy diagrams to arbitrary order. Now that I see that this changes the mass and charge of the involved particles, it seems incorrect to assume that even a single particle propagates as a free particle as there is always the vacuum which is not empty at all.

In order to compute the S-matrix elements correctly we would have to include all the diagrams with external leg corrections as well. This would increase the number of diagrams significantly. It would give the right result to the given order, right? It is just a lot work that we would like to get around. So this is where the LSZ reduction formula comes into play which relates the amputated diagram to the S-matrix element with additional factors, the renormalization factors Z. Therefore we can still just compute the amputated diagrams and add some pre-computed factors Z to the final result. This result will have time evolution with the interacting time evolution operator built in. And the time evolution operator is just another word for the propagators, right? The propagators obtain a correction like Σ or Π as we have computed in class and in the exercises.

Those corrections to the propagator are a bit hard to manage and we do not need to carry the Σ and Π around explicitly. The in- and out-states that we want to use with the S-matrix are on-shell particles which fulfil the Dirac equation $[\not{\boldsymbol{p}} - m]u(\boldsymbol{p}) = 0$ with the physical mass m. Therefore the fermion propagator $\mathrm{i}/[\not{\boldsymbol{p}} - m]$ will have a pole at the physical mass. The behaviour at the pole is characterized by the residue at the pole. Therefore we can just encapsulate the difference in the free and interacting propagator *for on-shell particles* in the residue of the propagator at the physical mass. The whole time evolution from the past infinity to the scattering event occurs with a single particle which has to me on-shell. Therefore the amputated diagrams is just multiplied with the residue of the pole. And that residue is called Z (or $\sqrt{Z}$?) and hence enters with LSZ formula.

All this means that we can continue as usual: Compute the amputated Feynman diagram using the various rules and tricks. Only at the end we tack on a number of Z factors to accommodate the corrections to the propagator for incoming and outgoing particles. The propagators in the amputated diagram are not corrected as we compute the diagram to a given order using the normal propagators. Using the corrected propagators there as well would shift the order of the diagram to a higher value which we do not necessarily want.

Peskin and Schroeder (1995) have Chapter 7 about all those ideas. There are Z and Z_1, Z_2 popping up here and there. Is the Z the one and only renormalization factor for the scalar ϕ^4-theory as there is only one kind of particle? Then the Z_1 seem to come from the vertex diagram where the transition from $\boldsymbol{\gamma}^\mu$ to $\boldsymbol{\Gamma}^\mu$ is done. Then there is Z_2 from the self-energy of the electron. In the end they turn out to be the same anyway in order to keep the photon massless. Then I think I saw a Z_3 somewhere as well, that was the correction to the electric charge from the self-energy of the photon which we have computed in the homework, right? The notation also gets a tad overloaded by the use of Σ for the exact result and Σ_2 as a second order result. Are any of the Z_2 or Z_1 just a second or first order approximation to the Z or distinct quantities? In the lecture there were Z_2^1 and similar constructs, so I assume that Z_2 and Z_1 are distinct as the book also suggests.

1 Yukawa corrections

The contribution to Z_1 was second in the book, the first renormalization constant introduced was Z and Z_2. The way they are introduced they are the probability to create a particular one-particle state from the vacuum. Annihilation is the same as the modulus squared is taken. That Z_2 was just added to the left side of the equation which relates the vertex $\boldsymbol{\Gamma}^\mu$ to the form factors F_1 and F_2 (ibid., (7.46)).

We need $F_1(0) = 1$ to make the theory consistent. Currently I think that this needs to be fulfilled to give the Ward identity which is equivalent to a massless photon? The problem with adding Z_2 on the left side is that $F_1(0)$ is not necessarily unity afterwards. To remedy this one introduced a Z_1 such that it would cancel the Z_2 in the case of $\boldsymbol{q} = 0$, the case of no momentum transfered (Peskin and Schroeder 1995, (7.47)).

Since the exact $\boldsymbol{\Gamma}^\mu$ and the exact Z_2 and Z_1 depend on all orders of α, we cannot compute them here. They are different though, since the vertex correction diagram contains a different propagator with mass and a different vertex without a Dirac matrix. This will alter the overall invariant matrix element of that vertex correction diagram in first order. Higher orders will be different as well. Corrections to the electron propagator with virtual ϕ-particles will also differ from the corrections with photons.

Perhaps we are just asked to verify the whole thing to second order in λ^2 which is the first interesting order? Then the computation of the first order vertex diagram would suffice and we could compare them to lowest order at least. Computing either diagram with dimensional regularization should be doable, especially since there are the Dirac matrices only from the electron propagators and not also from the electron-photon vertices.

The Z_2 is defined as the residue of the pole in the electron propagator (ibid., p. 243), therefore we should be able to compute that from the geometric series of first-order self-energy diagrams. Here I would proceed as we did with the self-energy of the electron with the photon. The 1PI-diagram could be computed, perhaps dimensional regularization is needed there. Then the expression Σ_2 (or was it Π_2?) can be put into the geometric series. Hopefully it would simplify enough that it can be evaluated. Then the residue would give me Z_2.

I still have no idea how to compute Z_1. It is introduced as a "second rescaling factor" (ibid., p. 230) which has to be equal to Z_2 such that it works. They refer to the proof in section 7.4 which is exactly where the Ward-Takahashi identity is proven. From the problem statement it sounds like one should be able to extract it from the vertex diagram.

However I fail to see where I could really start. Now my time runs out and I cannot do a Wick rotation and do this homework in imaginary time. Therefore I just have to leave you with three pages of questions. Sorry. Having read the sections 7.1–7.4 a couple times now I still have no clear idea how all these concepts relate. I would greatly appreciate if you could give me a few pointers into the right directions to clear it up.

References

Peskin, Michael E. and Daniel V. Schroeder (1995). *An Introduction to Quantum Field Theory*. Westview Press. ISBN: 978-0-201-50397-5.