Disclaimer

This is a problem set (as turned in) for the module physics615.

This problem set is not reviewed by a tutor. This is just what I have turned in.

All problem sets for this module can be found at
http://martin-ueding.de/de/university/msc_physics/physics615/.

[disclaimer]

TPP sheet 13 Martin Ueding

1. Gauge invariance in the Standard Model

Part (a)

The vev is a Lorentz scalar and globally the same.

Part (b)

$$\exp\left(i \frac{\alpha_a}{2} \tau_a\right) = \underbrace{\mathbb{1}}_{} + \underbrace{\frac{i\alpha_a}{2}\tau_a}_{} - \underbrace{\left(\frac{\alpha_a}{2}\right)^2 \mathbb{1}}_{} - \underbrace{\frac{i}{2}\left(\frac{\alpha_a}{2}\right)^3 \tau_a}_{} + \dots$$

$$\cos\left(\frac{\alpha_a}{2}\right)\mathbb{1} \qquad \tau_a \, i \sin\left(\frac{\alpha_a}{2}\right)$$

We need $\begin{pmatrix} ? & 1 \\ ? & 0 \end{pmatrix}$ as the transformation matrix. So $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is unitary.

$\tau_2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}$. Then $\alpha_2 = \pi$, $\alpha_1 = \alpha_3 = 0$.

Part (c)

Well, $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \ell^- \end{pmatrix} = \begin{pmatrix} \ell^- \\ -\nu_e \end{pmatrix} = \begin{pmatrix} \nu_\ell' \\ \ell' \end{pmatrix}$

Part (d)

$u = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

On homework 9 I do not find anything about that transformation of $F^{\mu\nu}$.

Each W_i - boson has a $F^{\mu\nu}$, just like each gluon.

We now have three types of indices:

- $\mu, \nu \in \{0,1,2,3\}$ are Lorentz indices
- $a, b, c \in \{1,2,3\}$ are $SU(2)$ algebra indices
- $i, j, k, l \in \{1,2\}$ are spin $\frac{1}{2}$ representation indices.

The transformation is:

$$F'^{\mu\nu}_a (\tau_a)^i{}_j = U^i{}_k F^{\mu\nu}_a (\tau_a)^k{}_l [U^{-1}]^l{}_j$$

Pull out F.

$$\rightsquigarrow F^{\mu\nu}_a \; i\tau_2 \; \tau_a(-i\tau_2)$$

$$= F^{\mu\nu}_a \; \tau_2 \tau_a \tau_2$$

$a = 2 \Rightarrow$ nothing happens

$a = 1 \Rightarrow \tau_2 \tau_1 \tau_2 = -i\tau_3 \tau_2 = -\tau_1$

$a = 3 \Rightarrow \tau_2 \tau_3 \tau_2 = i\tau_1 \tau_2 = i^2 \tau_3 = -\tau_3$

Therefore we get a minus for W_1 and W_3.

The $W^\pm$ is $\frac{1}{\sqrt{2}}[W_1 \pm iW_2]$

The transformation then gives

$$W^\pm \longrightarrow \frac{1}{\sqrt{2}}[-W_1 \pm iW_2] = -\frac{1}{\sqrt{2}}[W_1 \mp iW_2] = -W^\mp.$$

Part (e)

$$\mathcal{L}_{int} = \frac{g}{2} (\bar{L}_i)_\alpha \sum_a W_a^\mu (\tau_a)^i{}_j (\gamma_\mu)^\alpha{}_\beta \frac{\delta^\beta{}_\gamma - (\gamma_5)^\beta{}_\gamma}{2} (L^j)^\gamma$$

a: $SU(2)_w$ algebra; Flavor index
i,j: $SU(2)_w$ representation; Flavor index
μ: $SO(1,3)$ representation; Lorentz index
α,β,γ: $SO(1,3)$ representation; Dirac index

The Dirac structure is not of interest here.

$$\bar{L}_i \sum_a W_a (\tau_a)^i{}_j L_j$$

Transformation would do $L \to i\tau_2 L$

and $\bar{L} \to \bar{L}(i\tau_2)^\dagger = -i\bar{L}\tau_2^\dagger = -i\bar{L}\tau_2$

So: $L \tau_2 \sum_a W_a \tau_a \tau_2 L$

$\tau_2 \tau_a \tau_2$ gives a negative sign for $a=1$ and $a=3$. But we also get a negative sign from W_a for those indices. No overall change $\Leftrightarrow$ invariant.

Part (f)

Start with $\frac{g^2}{4}\left(\sum_a W_a^\mu \tau_a \varphi\right)^\dagger \sum_b W_b^\mu \tau_b \varphi$

$\varphi \to i\tau_2 \varphi$

$\rightsquigarrow \frac{g^2}{4}\left(i\sum_a W_a^\mu \tau_a \tau_2 \varphi\right)^\dagger i\sum_b W_b^\mu \tau_b \tau_2 \varphi$

We have $i^\dagger = -i$. Those cancel.

$$\sum_{a,b} \tau_2^\dagger \tau_a^\dagger \tau_b \tau_2 = -\sum_{a,b} \tau_2 \tau_a^\dagger \tau_b \tau_2$$

Insert $\tau_2^2 = \mathbb{1}$

$$= - \sum_{a,b} \underbrace{\tau_2 \tau_a^\dagger \tau_2}_{\substack{\text{Minus for} \\ a \neq 2}} \underbrace{\tau_2 \tau_b \tau_2}_{\substack{\text{Minus for} \\ b \neq 2}}$$

But we also get minus signs from W_a and W_b in those cases. That expression is also invariant.

Part (g)

That matrix would have to be:

$$U \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} v/2 \\ v/2 \end{pmatrix}$$

$$U = \begin{pmatrix} a/2 & 1/2 \\ b/2 & 1/2 \end{pmatrix}$$

We want $\det(U) = 1$, so $a = b$

$$U = \begin{pmatrix} a/2 & 1/2 \\ a/2 & 1/2 \end{pmatrix}$$

Also need $U^\dagger U = \mathbb{1}$

$$\frac{1}{4} \begin{pmatrix} 1 & 1 \\ a^* & a^* \end{pmatrix} \begin{pmatrix} a & 1 \\ a & 1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 2a & 1 \\ 2|a|^2 & 2a^* \end{pmatrix} \neq \mathbb{1}$$

So that does not exist.