Disclaimer

This is a problem set (as turned in) for the module physics615.

This problem set is not reviewed by a tutor. This is just what I have turned in.

All problem sets for this module can be found at
http://martin-ueding.de/de/university/msc_physics/physics615/.

[disclaimer]

TPP 1-1 Lorentz invariants

Part 1

$dx\ dy\ dz$ is volume 3-form.

Use transformation $x' = \varphi(x)$. 3-form transforms with pushforward. Just Jacobi determinant.

Only spatial part is taken into account.

$$dx\ dy\ dz = \varepsilon_{ijk}\, dx^i \wedge dx^j \wedge dx^k$$

$$dx'\,dy'\,dz' = \varepsilon_{ijk}\, \Lambda^i_1\, \Lambda^j_2\, \Lambda^k_3 \quad dx\,dy\,dz$$

$$= \varepsilon_{ijk}\, \Lambda^i_a\, \Lambda^j_b\, \Lambda^k_c\ \varepsilon^{abc}\ dV$$

$$= \det(\text{spatial part of } \Lambda) = \gamma\, dV$$

Part 2

Same thing, $\det(\Lambda) = 1$. Invariant.

Part 3

p^2 is invariant. $\delta^{(1)}$ is just a function, not affected.

Part 4

$\int \frac{d^3p}{2E}$ is invariant.

$d^3p \mapsto \gamma\, d^3p$

$E = p^0 \mapsto \gamma\, p^0$

Factor of γ cancels

TPP 01-2

Part 1

Klein-Gordon field. φ^* and φ are independent.

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial \varphi^*_{,\mu}} - \frac{\partial \mathcal{L}}{\partial \varphi^*} = 0$$

$$\partial_\mu \varphi^{,\mu} - m^2 \varphi = 0$$

$$[\Box - m^2]\varphi = 0$$ Klein-Gordon equation.

Similarly there is $[\Box - m^2]\varphi^* = 0.$

Part 2

$$\partial_\mu e^{i\alpha} = 0. \qquad [e^{i\alpha}]^* = e^{-i\alpha}$$

$|e^{i\alpha}| = 1.$ Invariant.

Part 3

Lagrangian does not change.

$$\varphi \mapsto \varphi + i\alpha\varphi \qquad \delta\varphi = i$$

$$j^\mu = \frac{\partial \mathcal{L}}{\partial \varphi_{,\mu}} \delta\varphi + c.c.$$

$$= \varphi^{*,\mu} i + c.c$$

$$j = i\left[\nabla\varphi^* - \nabla\varphi\right]$$

Part 4

j conserved $\Longleftrightarrow$ $j^\mu{}_{,\mu} = 0$

$$j^\mu{}_{,\mu} = i\left[\Box\varphi^* - \Box\varphi\right] = 0$$ due to e.o.m.

TPP 01-3

Part 1

$$S = -\frac{1}{4}\int d^4x \; F \wedge * F \qquad\qquad F = dA$$

What are the fields? $F^{\mu\nu}$? No. A_μ.

I want $dF = 0$

$$\mathcal{L} = -\frac{1}{4}\, dA \wedge * dA$$

$$d\,\frac{\partial \mathcal{L}}{dA} = d * dA = d * F = 0$$

And $dF = 0$.

Is that easy to see? Does that even work?

$$\mathcal{L} = -\frac{1}{4}\,\partial_{[\mu} A_{\nu]}\,\partial^{[\mu} A^{\nu]}$$

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial A_{\nu,\mu}} = \partial_\mu\, 2 A^{\nu,\mu} = 2\,\Box A^\nu$$

And swapped ones: $\quad 2\,\partial_\mu \partial_\nu A^\nu$

$$\rightsquigarrow d * dA$$

Writing down the equations for E and B is a waste of time.

Part 2.

$$\varepsilon = T^{00} \qquad \text{and} \qquad S^i = T^{0i}.$$

$$T^{00} = \frac{\partial \mathcal{L}}{\partial A_{\lambda,0}} A_\lambda{}^{,0} - \mathcal{L} g^{00}$$

$$= -\frac{1}{4}\left[A^{\lambda,0} - A^{0,\lambda}\right] A_\lambda{}^{,0} + \frac{1}{4} F \wedge * F$$

$$= \frac{1}{4}\Big[A^{0,\lambda}\dot{A}_\lambda - \dot{A}^\lambda \dot{A}_\lambda + \underbrace{F \wedge * F}_{2[E^2+B^2],\ \text{right?}}\Big]$$

$$K^{\lambda\mu\nu} = F^{\mu\lambda} A^\nu \qquad \partial_\lambda K^{\lambda\mu\nu} = \partial_\lambda F^{\mu\lambda} A^\nu$$

$$= \underbrace{F^{\mu\lambda}{}_{,\lambda}}_{=0} A^\nu + F^{\mu\lambda} A^\nu{}_{,\lambda}$$

$$F^{\mu\lambda} A^\nu{}_{,\lambda} = \left[A^{\lambda,\mu} - A^{\mu,\lambda}\right] A^\nu{}_{,\lambda} \overset{\mu=\nu=0}{\rightsquigarrow} \dot{A}^\lambda A^0{}_{,\lambda} - A^{0,\lambda} A^0{}_{,\lambda}$$

$$\partial_\mu \partial_\lambda F^{\mu\lambda} A^\nu = 0. \quad \text{uh!}$$

Start again.

$$\partial_\lambda K^{\lambda\mu\nu} = \partial_\lambda F^{\lambda\mu} A^\nu = F^{\lambda\mu}{}_{,\lambda} A^\nu + F^{\lambda\mu} A^\nu{}_{,\lambda}$$

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial A_{\tau,\mu}} A_\lambda{}^{,\nu} - \mathcal{L} g^{\mu\nu}$$

$$\mathcal{L} = -\frac{1}{4} A_{[\nu,\mu]} A^{[\nu,\mu]} = -\frac{1}{2}\left[A_{\nu,\mu} A^{\nu,\mu} - A_{\nu,\mu} A^{\mu,\nu}\right]$$

$$\frac{\partial \mathcal{L}}{\partial A_{\lambda,\mu}} = -\frac{1}{2}\left[A^{\lambda,\mu} - A^{\mu,\lambda}\right]$$

$$T^{\mu\nu} = -\frac{1}{2}\left[A^{\lambda,\mu} - A^{\mu,\lambda}\right] A_{\lambda}{}^{,\nu} + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} g^{\mu\nu}$$

$$\hat{T}^{\mu\nu} = -\frac{1}{2}\left[A^{\lambda,\mu} - A^{\mu,\lambda}\right] A_{\lambda}{}^{,\nu} + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} g^{\mu\nu}$$

$$+ \underbrace{F^{\lambda\mu}{}_{,\lambda}}_{=0?} A^{\nu} + \underbrace{F^{\lambda\mu} A^{\nu}{}_{,\lambda}}$$

$$\left[A^{\mu,\lambda} - A^{\lambda,\mu}\right] A^{\nu}{}_{,\lambda}$$

$$\hat{T}^{\mu\nu} = \frac{1}{2}\left[A^{\lambda,\mu} - A^{\mu,\lambda}\right] A_{\lambda}{}^{,\nu} + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} g^{\mu\nu}$$

$$\hat{T}^{00} = \frac{1}{2}\left[A^{\lambda,0} - A^{0,\lambda}\right] A_{\lambda}{}^{,0} + \frac{1}{4} \underbrace{F_{\alpha\beta} F^{\alpha\beta}}_{2(B^2+E^2)}$$

$$S^{i} = \hat{T}^{0i} = \frac{1}{2}\left[A^{\lambda,0} - A^{0,\lambda}\right] A_{\lambda}{}^{,i} \cancel{+ \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta}} g^{\mu\nu}$$

$$= \frac{1}{2} A^{\lambda,0} A_{\lambda}{}^{,i} - A^{0,\lambda} A_{\lambda}{}^{,i}$$