Anwesenheitsübung 7

2012-05-22

1) Friedmann Gleichung

1.

$$H(a)^2 = \frac{\dot{a}^2}{a^2} = \frac{(da)^2}{(dt)^2 a^2} = H(a)^2$$

$$\frac{da}{dt\, a} = H(a)$$ Dann Trennung der Variablen.

Für $\Omega_r = \Omega_0 = 1$

$$\int_0^{t'} dt = \int_0^1 \frac{da\, a^2}{a H_0} = \frac{1}{H_0} \int_0^1 a\, da$$

$$t' = \frac{1}{2H_0} = 6{,}7\ \text{Gyr}$$

2.

$$\int_0^{t'} dt = \int_0^1 \frac{da}{a H_0} \cdot \left(\frac{1}{a^3}\right)^{-\frac{1}{2}} = \frac{1}{H_0} \int_0^1 a^{\frac{1}{2}}\, da$$

$$= \frac{1}{H_0} \frac{2}{3} a^{\frac{3}{2}}$$

$$t' = \frac{2}{3H_0} = 8{,}9\ \text{Gyr}$$

2) Homogene Expansion

Strahlung $a(t) = \alpha\ t^{1/2}$

$T \propto t^{-1/2}$

$z = \frac{1}{c \cdot t^{1/2}} - 1$

$\boxed{T \propto \frac{1}{a}}$

$\boxed{z = \frac{1}{a} - 1}$

Materie: $a \propto t^{2/3}$

$T \propto t^{-2/3}$

Dunkle-Energie: $a \propto e^{t}$

$T \propto e^{-t}$

2.

$\dot{a}$:	$\frac{1}{2} t^{-\frac{1}{2}}$	$\frac{2}{3} t^{-\frac{1}{3}}$	e^t
$\ddot{a}$:	$-\frac{1}{4} t^{-\frac{3}{2}}$	$-\frac{2}{9} t^{-\frac{4}{3}}$	e^t
$H(t) = \frac{\dot{a}}{a}$	$\frac{1}{2} t^{-\frac{1}{2}-\frac{1}{2}}$ $= \frac{1}{2} t^{-1}$	$\frac{2}{3} t^{-\frac{1}{3}-\frac{2}{3}}$ $= \frac{2}{3} t^{-t}$	$H(t) \propto 1$

3) Hubble Constant

$z = 0.0231$

$$cz = H_0 D$$

$$\frac{cz}{H_0} = D \qquad D = 94{,}9 \ M_{pc}$$